Blind Self-Calibration of Sensor Arrays

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***Abstract*—In this presentation, I discuss several techniques that could start the self-calibration process without requiring a sky model. These techniques either exploit redundancy in the array configuration or aim to optimize a measure of image quality. In this context, image quality is usually measured in terms of image contrast or Shannon entropy. After a brief introduction of these techniques, I comment on their applicability in future radio telescopes and present open questions, which is particularly relevant in the context of development of the Square Kilometre Array (SKA).**

I. Introduction

The performance of many self-calibration techniques heavily depends on the quality of the initial estimate for the observed scene. This is equally true for the computational performance and the estimation performance. In radio astronomy, much effort is therefore invested in building sky models, that can be used to start the self-calibration process. In this paper, I discuss two blind self-calibration techniques, redundancy calibration and calibration based on image optimization. The key selling point of blind techniques is that they do not require prior knowledge on the observed scene. I present these methods in the radio astronomy context and comment on their potential use in future radio telescopes like the Square Kilometre Array (SKA) [**?**].

II. Redundancy calibration

The key idea behind redundancy calibration is that the correlation of signals from pairs of antennas with identical baseline vectors measure the same spatial frequencies, regardless of the observed brightness distribution [**?**]. A baseline vector represents the difference between two antenna positions. If the correlations measured on two identical baselines differ, this is caused by instrumental artefacts. If we can assume that all antennas have the same directional response and that imperfections in the analog electronics have a negligible impact on the measured correlations, we can use the principle of redundancy calibration to calibrate the antenna gain and phase.

Placement accuracy is an important consideration in the context of redundancy calibration to ensure sufficient identicalness of the redundant baselines. What identicalness is “sufficient”, depends on the required calibration accuracy. This provides some room to apply redundancy calibration to baselines that are only approximately redundant. We can demonstrate that this can be exploited in initial calibration in relatively dense irregular arrays [**?**]. This initial calibration could be used as starting point for a normal self-calibration process instead of a sky mnodel.

To exploit redundancy in sparse irregular configurations, redundant baselines probably have to be designed into the system. Since redundant baselines observe the same spatial frequencies, they do not provide additional information on the brightness distribution. An array configuration without redundant baselines may therefore be more attractive from an imaging perspective. Can we provide guidelines to make a trade-off between facilitating the calibration process or facilitating the imaging process?

III. Image optimization

In the absence of noise, the vectorized measured covariance matrix is described by

$$\mathrm{vec}\left(\widehat{\mathbf{R}}\right) = \mathrm{diag}\left(\overline{\mathbf{g}} \otimes \mathbf{g}\right)\mathbf{M}\boldsymbol{\sigma} = \mathbf{M}_{\mathbf{g}}\boldsymbol{\sigma}, \tag{1}$$

where $\mathbf{g}$ is a vector containing the direction independent sensor gains, $\mathbf{M}$ is the measurement matrix for a perfectly calibrated instrument, $\boldsymbol{\sigma}$ is a vector containing image parameters, e.g., wavelet coefficients or pixel values, $\otimes$ denotes the Kronecker product and $\mathrm{diag}(\cdot)$ forms a square matrix with its vector argument on the main diagonal. In the imaging process, (**??**) is inverted. Assuming that $\mathbf{M}_{\mathbf{g}}$ is invertible, this gives $\boldsymbol{\sigma} = \mathbf{M}_{\mathbf{g}}^{-1} = \mathrm{vec}\left(\widehat{\mathbf{R}}\right)$. Since $\mathbf{M}_{\mathbf{g}}$ depends on the gains, the image can be optimized by finding the right gain values.

What constitutes an optimal image is still a matter of debate. Contrast optimization is a very intuitive choice that can, for example, be formulated as

$$\widehat{\mathbf{g}} = \underset{\mathbf{g}}{\mathrm{argmax}} \frac{1}{\langle\boldsymbol{\sigma}\rangle}\sqrt{\left\langle\left(\boldsymbol{\sigma} - \langle\boldsymbol{\sigma}\rangle\right)^2\right\rangle}, \tag{2}$$

where $\langle\cdot\rangle$ denotes the averaging operator. Other measures for image contrast include maximization of the peak value in the image (with an appropriate constraint on the gain amplitudes, of course). However, the measure given in (**??**) seems to be more robust in scenes with multiple sources [**?**].

Another, but less developed idea is to maximize the Shannon entropy of the image S given by [**?**]

$$S = -\sum_i \frac{|\sigma_i|^2}{\|\boldsymbol{\sigma}\|^2}\log\frac{|\sigma_i|^2}{\|\boldsymbol{\sigma}\|^2}. \tag{3}$$

It has been demonstrated that maximization of the Shannon entropy can be used for phase calibration [**?**], but does it also allow gain amplitude calibration? Also, more analysis is required to assess the calibration quality achievable with entropy maximization. This need not be a problem as long as it provides sufficient gain calibration accuracy to make a first image that can than be improved using self-calibration cycles.