Application of Redundancy Calibration to Phased Arrays and Some Limitations

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Abstract

In new phased array instruments, a fundamental question is whether the geometrically redundant baselines in a regularly arranged phased array are really redundant. Based on real and simulated data, we demonstrate that for a phased array station, a regular arrangement of station elements is necessary but not sufficient to satisfy redundancy calibration requirements. This is due to the electromagnetic interaction between closely spaced antenna elements which leads to non-identical beams and possibly correlated noise. Each of these deterministic effects introduces a bias on the estimated calibration results. Understanding the nature of these effects has helped us to determine the limits of applicability of redundancy calibration for a given phased array, which are demonstrated in this paper.

1 Introduction

The Low Frequency Array (LOFAR) [1] and the Electronic Multi-Beam Radio Astronomy Concept (EMBRACE) [2] are two examples of phased array instruments built on the technology roadmap towards the Square Kilometre Array (SKA), a future radio telescope envisaged to be at least an order of magnitude more sensitive than current radio telescopes [3]. Figure 1 shows that the LOFAR HBA stations and EMBRACE have a regular antenna placement, which creates many redundant baselines, i.e., baselines with the same physical length and orientation. Noordam and De Bruyn have shown how using redundant information of the WSRT (Westerbork Synthesis Radio Telescope) significantly improves the fidelity of its radio image [4], while Wieringa presented a linear formalism for redundancy calibration by taking the natural logarithm of the visibilities [5]. Its proven efficiency, sky model independence, linearity and low computational cost, motivated us to consider its application to new radio telescopes, in particular at stations where there is geometric redundancy. This work began with [6] which presents preliminary results of redundancy calibration using real data of an HBA station. Redundancy calibration has been comprehensively re-investigated for phased arrays in [7]. Following the work in [6] and [7], we briefly present what causes bias in redundancy calibration results and under which circumstances these limit the calibration solutions.

In phased array stations, the output of each station element is correlated to the others to provide station visibilities. Redundancy calibration makes use of all redundant information in the measured visibilities to calibrate for the station element gains. It is based on the assumption that we should capture the same true visibility on redundant baselines. This is feasible, if the station elements have identical beams. For closely spaced elements in a phased array station, such as the ones shown in Fig. 1, mutual coupling between them may lead to non-identical element beams and correlated noise. Consequently, they cause limitations to the station calibration. Disentangling these effects before calibration is possible under certain conditions, but it is tedious in practice. Thus, we investigate the applicability and accuracy of the redundancy calibration in their presence.

2 Station data model and redundancy calibration formalism

In phased array stations, correlations between all station elements can be calculated. These correlations are called station visibilities which are used for engineering purposes e.g. station calibration and RFI detection

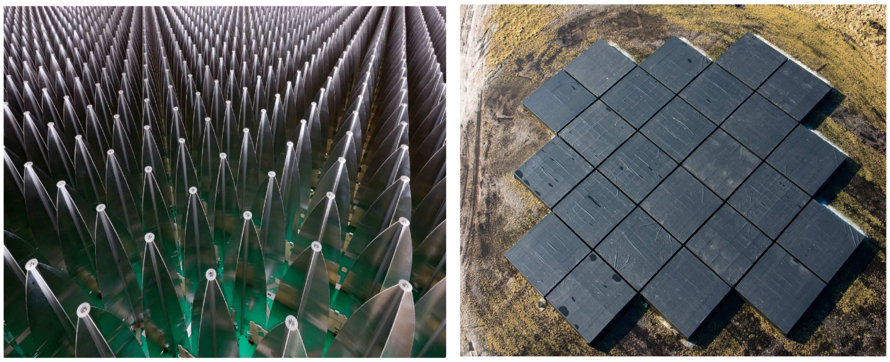

Figure 1: Left: An EMBRACE station. The output signals of 72 dual-polarized dipoles are coherently summed to form a tile signal. Right: A 24-tile HBA station. The output signals of 16 dual-polarized dipoles are coherently summed to form a tile signal. Stations like these are highly redundant.

and mitigation. Let us assume that we have a station of P elements and a perfect correlator without any offset. Then, the general model for the station visibility matrix can be written as:

$$\mathbf{R} = \mathbf{G}(\mathbf{M}\mathbf{A}\boldsymbol{\Sigma}_{\mathrm{s}}\mathbf{A}^{H}\mathbf{M}^{H})\mathbf{G}^{H} + \boldsymbol{\Sigma}_{\mathrm{n}} \tag{1}$$

where $\boldsymbol{\Sigma}_{\mathrm{s}}$ $(Q \times Q)$ is a diagonal matrix containing the flux, σ_q, of Q mutually independent i.i.d. Gaussian signals, impinging on the array. The signals are assumed to be narrowband, so we can define the Q array response vectors $\mathbf{a}_q$ which include the phase delays due to the geometry and the directional response of the array. $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_Q]$ $(P \times Q)$ is a stack of the array response vectors. $\mathbf{M}$ $(P \times P)$ expresses mutual coupling. $\mathbf{G}$ $(P \times P)$ is the diagonal matrix of the element complex gains, g_i. $\boldsymbol{\Sigma}_{\mathrm{n}}$ is the noise covariance matrix, which describes the, possibly correlated, noise in the system and crosstalk effects and is generally a non-diagonal matrix.

To build up the system of equations for the redundancy calibration algorithm, we study an off-diagonal element of $\mathbf{R}$ in (1), i.e., a model of the visibility on baseline between the elements i and j, in more detail:

$$R_{ij} = g_i g_j^* \left[(\mathbf{M}\mathbf{A})\boldsymbol{\Sigma}_{\mathrm{s}}(\mathbf{M}\mathbf{A})^{H}\right]_{ij} + \Sigma_{\mathrm{n},ij}, \tag{2}$$

where $(\cdot)^*$ denotes complex conjugation. This can be expanded to

$$R_{ij} = \overbrace{g_i g_j^* \sum_{q=1}^{q} \left(\sum_{\substack{p_1=1 \\ p_1 \neq i}}^{p} \sum_{\substack{p_2=1 \\ p_2 \neq j}}^{p} M_{ip_1} M_{jp_2}^* A_{p_1 q} A_{p_2 q}^* \sigma_q \right)}^{\text{additive term due to mutual coupling}} + g_i g_j^* M_{ii} M_{jj}^* \underbrace{\sum_{q=1}^{q} A_{iq} \sigma_q A_{jq}^*}_{\text{true visibility}} + \Sigma_{\mathrm{n},ij} \tag{3}$$

Without loss of generality, we can take $M_{ii} = 1$. To show the analogy between (3) and the redundancy method formalism in [5], we define the following term:

$$e_{ij} = g_i g_j^* \sum_{q=1}^{q} \left(\sum_{\substack{p_1=1 \\ p_1 \neq i}}^{p} \sum_{\substack{p_2=1 \\ p_2 \neq j}}^{p} M_{ip_1} M_{jp_2}^* A_{p_1 q} A_{p_2 q}^* \sigma_q \right) + \Sigma_{\mathrm{n},ij} \tag{4}$$

and rewrite (3) as

$$R_{ij} = R_{ij}^{true} g_i g_j^* + e_{ij} \tag{5}$$

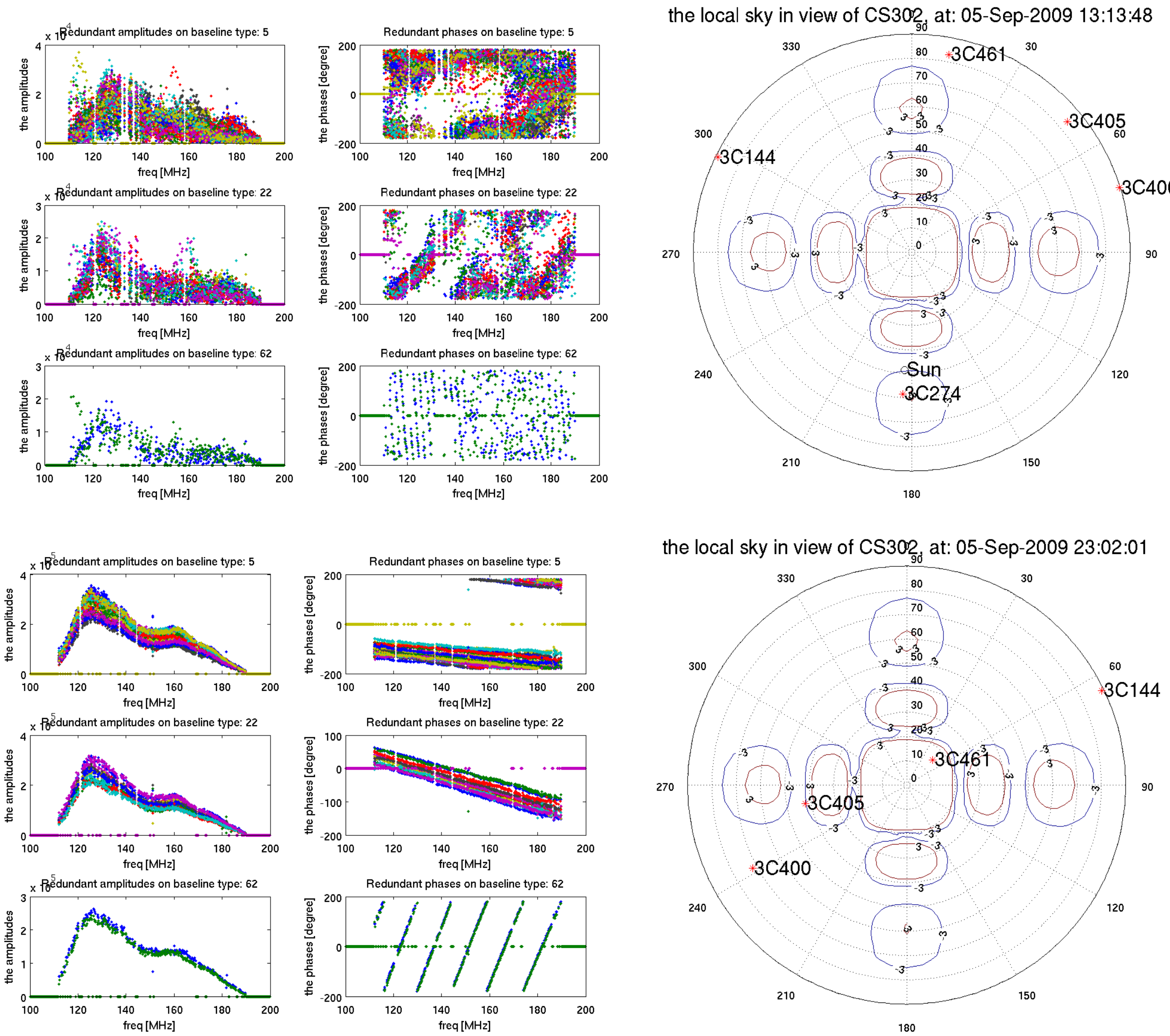

Figure 2: An observation with an HBA station on September 5, 2009 at 13:13:48 UTC in which redundancy calibration fails (top) and an observation with the same station at 23:02:01 in which redundancy calibration is successful (bottom). The left panels show the measured visibilities for three distinct type of redundant baselines. The right panels show the local sky model at the time of observation with the strongest sources on the sky superimposed on a contour plot of the element beam pattern at 170 MHz.

Redundancy calibration is most reliable when the baseline dependent error (e_{ij}) is negligible, as it was assumed in [5]. This effect has not been of concern for an array such as the WSRT whereas they are present in the data model for phased array stations, presented in (1). This term introduces inaccuracy in the calibration results. We refer the reader to [7] for quantitative analysis.

3 Verification of redundancy assumption for a HBA station

To verify the fundamental assumption of redundancy, we have checked the identicalness of tile beams in a 24-tile HBA station using CAESAR [8]. As expected, the results showed that the beams of the individual tiles in the station are identical only in their main lobe and differ in their sidelobes [7]. This means that redundancy holds if a strong source falls in main lobe such that, its signal dominates over the signal of other sources present in the sidelobes where the beams differ significantly. This has been confirmed by real observations presented in Fig. 2. The top panel shows that non-identical sidelobes break down the redundancy. The bottom panel shows that redundancy holds when there is a strong source in the main lobe whose signal dominates over the signals of other strong sources which have fallen in the non-identical sidelobes. The sidelobe levels are at least 25 dB below the main lobe in a HBA station.

4 Conclusion

To apply redundancy calibration to a station, we have to design for it. This means that it requires not only a regular arrangement of antenna elements but also an appropriate spacing between them such that the elements obtain identical beams. These considerations implicitly reduce the effect of baseline dependent error terms which affect the calibration accuracy. However with the current design of HBAs, the redundancy calibration is applicable by tracking a strong source. Its calibration results improve up to $\sim 0 - 1\%$ in phases and up to $\sim 0 - 2\%$ in amplitudes as compared to the model-based method [7]. Moreover it is computationally cheap, sky model independent, less sensitive to RFIs and simple to implement all of which are of importance for an extremely large radio telescope such as the Square Kilometer Array.

5 Acknowledgments

We would like to greatly thank Rob Maaskant and Michel Arts for their generous help with CAESAR simulations.

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