Calibratability by Design for SKA's Low Frequency Aperture Array

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Abstract

Early universe science with the Low Frequency Aperture Array (LFAA) Element of the Square Kilometre Array (SKA) requires the ability to produce thermal noise limited images after 1000 hours of integration. In this paper, we summarize the effects that determine the noise floor in such deep images and sketch the calibration strategy required to track variations in (ionospheric) propagation conditions and instrumental effects over an observation. Defining calibratability as the ability to produce deep thermal noise limited images, we show that calibratability imposes a number of constraints on the design of the array. We conclude that an array of small stations needs to meet a certain density of stations over the area covered by the array and that the antenna configuration within the individual stations should either have different randomizations or be rotated copies of the same randomized configuration.

1. Introduction

Tomography of the redshifted 21-cm signal from the Epoch of Reionization (EoR) and statistical analysis of the same signal from the Cosmic Dawn are likely to be the most challenging science goals of the Low Frequency Aperture Array (LFAA) element of the Square Kilometre Array (SKA) [1,2]. These goals require the LFAA system to be capable of producing very deep (1000 hours of integration) thermal noise limited images. The following factors determine the effective noise floor in deep images [3,4]

- Thermal noise;
- Classical source confusion;
- Calibration noise caused by extraction of information from the data to estimate calibration parameters, that is then no longer available to reconstruct the image;
- Calibration artefacts caused by biased calibration solutions, e.g., self-calibration bias [5];
- Point spread function (psf) sidelobe confusion noise (PSCN) caused by insufficient suppression of the sea of weak sources inside the field-of-view (FoV) [6, §5.2];
- Far sidelobe confusion noise (FSCN) caused by insufficient suppression of all not individually subtracted sources outside the FoV [6, §5.2.7];

Since the last five effects typically cannot be avoided, the noise floor in "thermal noise limited" maps will typically be a factor 1.5 – 2 higher than the thermal noise [7]. In this paper, we discuss the implications of avoiding calibration artefacts and of minimizing the impact of PSCN and FSCN on the design of the system.

Proper calibration of time-varying direction dependent effects (DDEs) is a crucial step to avoid calibration artefacts. Time-varying DDEs are caused by variations in (ionospheric) propagation conditions and in instrumental response. In the next section, we sketch a calibration strategy that can deal with such time-varying DDEs. This strategy puts clear time constraints on the update rate of certain calibration parameters. In section 3, we show that this leads to a requirement on the density of stations over the entire area covered by the synthesis array. The impact of PSCN and FSCN needs to be mitigated by ensuring sufficiently low psf and station sidelobe levels respectively. In sections 4 and 5, we show that this leads to a stringent requirement on the average station sidelobe level that can only be met by stations that either have differently randomized antenna configurations or are rotated copies of the same randomized antenna configuration. The conclusions are summarized in section 6.

2. Top level calibration strategy

Self-calibration is required to track time-varying DDEs caused by ionospheric propagation conditions and instrumental response. At LFAA operating frequencies (50 – 350 MHz), ionospheric conditions impose the most stringent constraints on calibration update rates. Turbulent ionospheric conditions may even be intractable for self-calibration because the update rates required become so short that the calibration sources inside the (FoV) cannot be observed with sufficient signal-to-noise ratio (SNR). A practically feasible system should be able to work with a short-term calibration update rate of order 10 seconds. This is usually sufficient to track traveling ionospheric disturbances (TIDs) but insufficient to track Kolmogorov turbulence [6, §4.3]. If the latter dominates the ionospheric behavior, observing conditions are too poor to produce high-quality data. Instrumental effects can cause DDEs as well, for example a nominal station beam model in amplitude and polarization characteristics needs updating at a typical timescale of order 10 minutes during sky tracking [6, §3.6.4]. Electronic gain calibration errors or failure of antenna elements in a station cause additional deviations from the nominal station beam but fluctuate at longer timescales.

These considerations naturally lead to a top level calibration strategy with a short-term update rate of 10 seconds and a mid-term update rate of 10 minutes. The short-term updates should take care of ionospheric variations in phase and Faraday rotation. Given the wide FoV, this requires direction dependent calibration towards five calibration sources to allow tracking of a parabolic ionospheric phase screen over the FoV [6, §4.8]. This holds particularly for the lowest operating frequencies (< 150 MHz). By integrating to timescales of 10 minutes while making appropriate corrections for ionospheric DDEs, another self-calibration process can improve the DDE model with an update rate of 10 minutes. With such integration times, the FoV is well-populated with sources with sufficient SNR to construct a web of tens of directions inside the FoV to build an accurate DDE model. This detailed model can be used to make direction dependent corrections while accumulating the data further. The complete data set may need to be processed twice or thrice to optimally exploit the detailed sky model constructed after full integration. The feasibility of a procedure like the one sketched above has been argued in [6, Ch. 4 and Ch. 5] and is demonstrated in deep imaging with LOFAR [7].

3. Minimum station density

The proposed LFAA station diameter of 35 m is envisaged to have 256 antennas giving full aperture filling below 110 MHz [1]. Assuming a parabolic taper over the aperture gives 75% aperture efficiency and a station beam width $\theta_0 = 1.28\,\lambda / D_{stat}$ (FWHM) as well as low side lobes. Taking a system temperature that is 20% higher than the sky noise we find a sensitivity per baseline of 5.9 Jy after 10 s integration over 1 MHz at 50 MHz. As a result, about 5.7 sources per beam have SNR > 3 per polarization, sufficient for self-calibration at a cadence of 10 s. The cross-section of the station beam at ionospheric height is larger than the typical ionospheric patch size and a more complicated model than a parabolic phase screen is required. This cannot be fitted due to lack of calibration sources in the average field on 10-s timescales. Due to the large number of stations (1024), the cross-sections of the station beams at ionosperic height are likely to have considerable overlap. If the density of piercing points of lines-of-sight from each station towards each calibration source is sufficiently high, a joint solution for all stations allows for accurate calibration of the ionospheric phase screen over the entire array.

The total area A_{ion} that needs piercing points at height h above an array with maximum baseline B_{max} between stations with diameter D_{stat} is given by $A_{ion} = (\pi/4)(B_{max} + h\beta\lambda/D_{stat})^2$. The typical patch size of a TID with wavelength λ_{TID} is $A_{patch} = (\pi/4)(0.2\lambda_{TID})^2$ and requires five piercing points for a second order TEC screen solution per patch. The total number of piercing points N is then given by $N = 5A_{ion}/A_{patch}$. If every station provides N_{cal} piercing points, this requires $N_{stat} = N / N_{cal}$ homogeneously distributed stations. Combining the expressions, we get

$$N_{stat} \geq \frac{125}{N_{cal}\lambda_{TID}^2}\left(\frac{h\beta\lambda}{D_{stat}} + B_{\max}\right)^2 \tag{1}$$

To see what this implies for the envisaged LFAA array ($D_{stat} = 35$ m, $B_{max} = 70$ km) [1], we compute the number of stations required to make the synthesis array calibratable at its lowest operating frequeny of 50 MHz ($\lambda = 6$ m), where the stations have sufficient sensitivity to track at least $N_{cal} = 5.7$ calibration sources. Assuming a parabolic weight taper over the filled station aperture, we take $\beta = 1.28$ and we assume a shortest $\lambda_{TID} = 90$ km [6, §4.3] and $h = 300$ km. We then find that ionospheric calibration requires at least 50 stations distributed homogeneously over the 70 km area. For a synthesis array with 1024 stations, this leaves 95% of collecting area to form the desired high-density core.

4. Array sidelobe level

A high psf sidelobe level may cause PSCN by insufficient suppression of all sources weaker than S_{max} inside the FoV [6, §5.2] after all sources stronger than S_{max} are subtracted. We can analyze the impact of PSCN by considering the RMS flux ΔS_{PSCN} over the FoV, which extends to an angle θ_0 from the pointing center. Assuming that the average gain over the FoV is half the peak gain, we find

$$\Delta S^2_{PSCN} = \int_0^{S_{max}} \int_0^{2\pi} \int_0^{\theta_0} \rho(S)(S/2)^2 E^2_{psf}(\theta,\phi) \sin\theta \, d\theta \, d\phi \, dS, \quad (2)$$

where $\rho(S)$ is the density of sources with flux S, and E_{psf} is the RMS sidelobe of the psf. Since the sources are randomly distributed, we can ignore the depedence of the RMS psf sidelobe level on direction while evaluating the integral. We then find

$$E_{psf} = \frac{1}{\sqrt{\pi}} \left(\frac{\Delta S}{\sin(\theta_0/2)\Delta S_m} \right), \quad (3)$$

where ΔS is the thermal noise level against which the PSCN needs to be balanced and ΔS_m^2 is the integrated squared source flux per steradian of all sources weaker than S_{max}.

Considering the LFAA sensitivity after integration over 1000 hours and 1 MHz, we find a thermal noise level of σ_I = 1.29 µJy in total intensity at 110 MHz. A spectral index α is used to convert flux S_f at frequency f to 1.4 GHz flux using $S = S_f(f/1.4)^{\alpha}$. Extrapolating the integrated 1.4 GHz source statistics from [6, §4.5] given by $N(>S) = 23.1\ S^{-1.33}$ in sources per steradian that exceed a flux S, we derive $\rho(S)$ by differentiation and ΔS_m by integration. We can now construct the following table with S_{max} at 110 MHz, α, S_m at 1.4 GHz, the number of sources per steradian and the number of sources per beam that have to be subtracted with flux larger than S_m, ΔS_m and E_{psf}.

S_{max}	α	S_m (µJy)	$N(>S_m)$	$N_B(>S_m)$	ΔS_m (mJy sr $^{-0.5}$)	E_{psf}
σ_I	0.7	0.22	1.65 10^{10}	1.3 10^8	40	
10 σ_I	0.7	2.17	7.86 10^8	6.1 10^6	76	3.2 10^{-5}
100 σ_I	0.6	28	2.62 10^7	2.0 10^5	198	1.2 10^{-5}

We see that the total number of sources over the full sky exceeding the noise level would be 2 10^{11}, comparable to the estimated number of galaxies in the universe. This is a warning about the used source count extrapolation. It also indicates that we are confusion limited by a longest baseline of 70 km providing a resolution of 8", while 1" would be needed to resolve all galaxies that have to be subtracted. The second row requires subtraction of six million sources, which does not seem feasible given the limited resolution. The third line indicates subtraction of 200,000 sources, which is feasible in principle given a longest baseline of 70 km, and a RMS psf sidelobe level of -49 dB. However, neither subtraction of 200,000 sources nor a -49 dB RMS psf sidelobe level have been demonstrated in practice to date.

Since a 1000-hour observation will consist of multiple shorter observations of, say, 6 hours, the side lobe level requirement should be met after every 6-hour observation since each partial observation will observe the field with (almost) the same array and source geometry. This requires a good (u,v)-coverage with a smooth variation in sampling density of the aperture. A sparse random array with N_{stat} stations has an instantaneous psf sidelobe RMS level of $1/N_{stat}$, in our case ~10^{-3}. When multiple independent snapshots are combined, this value can be reduced until the (u,v)-distribution is filled. Further reduction is then possible by appropriate weighting and tapering. We have assumed 1 MHz bandwidth, since that seems to be a reasonable spectral coverage for a single plane in EoR data analysis. We have picked 110 MHz, since the most stringent requirements are found near the frequency were the sensitivity peaks.

5. Station sidelobe level

A high station sidelobe level may cause FSCN by insufficient suppression of sources by the station sidelobes. Sources outside the FoV should be estimated and subtracted individually. Due to the limited amount of information available on the timescales on which the DDE estimates towards these sources need to be updated, this can only be done for a very limited number of sources, which we will set to 50 in our analysis. All other sources cannot be taken into account individually. The RMS flux of all these sources therefore has to be suppressed to the thermal noise level by the station sidelobes. This RMS flux can be described by the following equation, which is very similar to (3):

$$\Delta S^2_{FSCN} = \int_0^{S_{max}} \int_0^{2\pi} \int_{\theta_0}^{\pi/2} \rho(S) S^2 E^2_{stat} E^2_{psf} \sin\theta \, d\theta \, d\phi \, dS, \quad (4)$$

where E_{stat} is the RMS station sidelobe level. In this case, S_{max} is the flux of the brightest source anywhere outside the FoV that cannot be individually treated. Balancing the FSCN level against the thermal noise level, we find

$$E_{stat} = \frac{1}{\sqrt{2\pi}} \left(\frac{\Delta S}{E_{psf} \Delta S_m} \right). \tag{5}$$

For our example of the LFAA at 110 MHz after 1000 hours of integration over 1 MHz, we find S_{max} = 5.8 Jy (at 1.4 GHz) based on the source statistics in [6, §4.5], assuming that the 50 stronger sources are individually treated. This gives ΔS_m = 35 Jy $sr^{-0.5}$. Assuming a RMS psf sidelob level of -49 dB, (5) then indicates, that we require a station sidelobe level of about -47 dB. This level cannot be achieved with a single station, especially if we need to take element failures into account. However, if different stations have different configurations, a source in a sidelobe of one station may be located in a null of another station. As a result, that source will be effectively suppressed on the baseline formed by those two stations. This argument shows that we need to consider the average sidelobe pattern of all visibilities [8]. Since the sidelobe pattern of each visibility is the result of aperture sampling obtained by the convolution of the two station configurations involved, the average sidelobe pattern of all visibilities is the result of the average aperture sampling obtained by convolving all possible pairs of station configurations. If each station has a different randomized distribution of antennas, the average aperture sampling will resemble that of an (apodized) filled aperture. With such a sampling, we can easily achieve an average RMS station sidelobe level of -45 dB. For LOFAR, it has been demonstrated that even regular station configurations, which lead to grating lobes when scanning away from the zenith and at higher operating frequencies, can have very low average sidelobe level as long as the stations have different orientations [8]. However, whether grating lobes can be sufficiently suppressed by station rotation for the LFAA, needs to be validated in simulation.

6. Conclusions

If we define calibratability as the ability to produce deep thermal noise limited images, calibratability imposes a number of constraints on the design of a radio telescope array. These constraints should ensure that thermal noise dominates over effects like calibration artefacts, psf sidelobe confusion noise and far sidelobe confusion noise in deep observations. In this paper, we derived the minimum station density, psf sidelobe level and station sidelobe level for the LFAA Element of the SKA. We found that 50 stations should be homogeneously distributed over the 70 km synthesis aperture to handle typical ionospheric circumstances at the lowest operating freqeuncy of 50 MHz. We also found that the required sidelobe levels are such, that a typical 6 hour synthesis observation should have a completely filled (u,v)-distribution to allow for appropriate weighting and tapering. In addition we found that the stations should either be rotated copies of each other or have different random configurations, and use appropriate tapering. A proper design for calibratability of the LFAA Element should be based on the principles presented in this paper, but should also consider more complicated ionospheric models taking its 3-D structure into account and cover the full LFAA frequency range.

7. References

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