Blind Calibration of Dense Irregular Arrays by Exploiting Multiply-Measured Nearly-Identical Spatial Frequencies

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***Abstract*—In this paper, I introduce a novel method for blind calibration of dense irregular arrays on an arbitrary scene. This method exploits the fact that in such arrays, several pairs of antennas may sample almost the same spatial frequency. I demonstrate this method using actual data from a single aperture array station of the Low Frequency Array radio telescope. This example showcases the advantages of blind calibration over calibration based on a source model in actual operating conditions. The results also indicate that the impact of errors introduced by modeling nearly-identical spatial frequencies as being identical may be less than the impact of the commonly made (but wrong) assumption that all antennas have the same radiation pattern. The proposed method could thus be an interesting alternative for calibration of the very compact Low Frequency Aperture Array stations of the Square Kilometre Array.**

I. Introduction

The Low Frequency Aperture Array (LFAA) subsystem of the Square Kilometre Array (SKA) [1], which is currently under development by the radio astronomy community, is envisaged to consist of subarrays (called *stations*) consisting of 256 antennas in an irregular configuration with a diameter of only 35 m. In such a dense (meaning tightly-packed in the context of this paper) irregular array, many antenna pairs will sample nearly-identical spatial frequencies, i.e., the co-array of the station configuration will contain many points that almost coincide. In radio astronomy, the points in the co-array are referred to as *baselines* and multiply-measured baselines are referred to as *redundant baselines*. Multiply-measured spatial frequencies are commonly exploited in regular antenna arrays for DOA estimation and blind calibration [2]–[4]. In radio astronomy, the latter is referred to as *redundancy calibration* [5]. In the next section, I introduce a novel method for blind calibration of dense irregular arrays by exploiting multiply-measured nearly-identical spatial frequencies, i.e., nearly-redundant baselines. I also quantify what constitutes a nearly-redundant baseline. In Sec. III, I illustrate the proposed method by applying it to actual data from a station of the Low Frequency Array (LOFAR) [6], an operational pathfinder for SKA in Europe, and discuss the impact of the modeling error introduced in near-redundancy calibration compared to the impact of the commonly made (but wrong) assumption that all antennas in the array have the same radiation pattern.

II. Near-redundancy calibration

In self-calibration or auto-calibration problems, the model for the array covariance matrix of the antenna signals can generically be formulated as

$$\mathbf{R} = \mathbf{G}\mathbf{R}_0\left(\boldsymbol{\theta}\right)\mathbf{G}^H + \boldsymbol{\Sigma}_n, \tag{1}$$

where $\mathbf{G}$ is a diagonal matrix with the unknown receiver path gains on the main diagonal, $\mathbf{R}_0$ is the expected covariance matrix response for a perfectly calibrated noise-free array parameterised by the parameters in the vector $\boldsymbol{\theta}$ and $\boldsymbol{\Sigma}_n$ is the noise covariance matrix, which I will assume to be diagonal. If there are (nearly-)redundant baselines in the array, the expected value of the corresponding entries of $\mathbf{R}_0\left(\boldsymbol{\theta}\right)$ will be (nearly-)identical. We can then formulate a selection matrix $\mathbf{I}_s$ such that

$$\mathrm{vec}\left(\mathbf{R}_0\right) = \mathbf{I}_s\boldsymbol{\theta}, \tag{2}$$

where $\mathrm{vec}\left(\cdot\right)$ denotes vectorisation of a matrix by stacking its columns into a single vector. If there is a sufficiently large number of (nearly-)redundant baselines, $\boldsymbol{\theta}$ and $\mathbf{G}$ can be estimated simultaneously [4].

What constitutes a (nearly-)redundant baseline can be defined by a maximum allowed deviation $\Delta x_{\max}$ from a specific point in the co-array. Assuming a uniform random distribution of points around the specified point in the co-array, the expected value of the RMS deviation within a circular area with radius $\Delta x_{\max}$ was found to be only $\Delta x_{\mathrm{rms}} = 0.24\Delta x_{\max}$. In the example presented in the next section, the worst case tolerance is $\Delta x_{\max} = 0.2\lambda$, where λ is the observed wavelength, resulting in $\Delta x_{\mathrm{rms}} = 0.05\lambda$. Such a $\lambda/20$ error is perfectly acceptable in many applications.

III. Application example

To demonstrate the effectiveness of the proposed method, I used data captured at a LOFAR station between 21:56:42 UTC and 22:05:13 UTC on June 25, 2010. The data consisted of array covariance matrices integrated over 1 s for each of the 512 195.3125-kHz wide frequency channels covering the frequency range from 0 to 100 MHz. The station consisted of a 46-antenna irregular array augmented with two calibration antennas placed about 65 m away from the array center. The long baselines to the calibration antennas are used to spatially

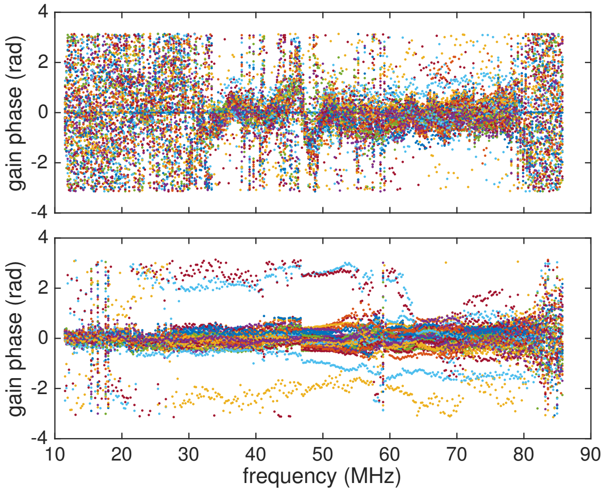

Fig. 1. Gain phase solutions obtained using conventional calibration on known sources (top) and using the proposed blind calibration method using nearly redundant baselines (bottom).

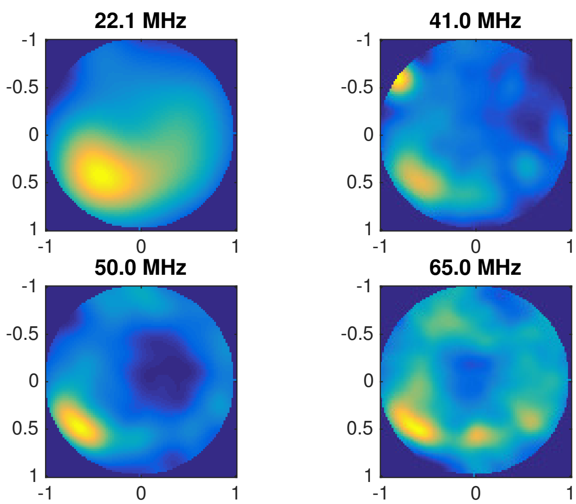

Fig. 2. All sky images at a few selected frequencies obtained after blind calibration using the proposed method.

filter out diffuse emission from the Galactic plane (as clearly seen in the sky images shown in Fig. 2), such that a simple point source model can be used for calibration.

The data were calibrated over the 11.5–85.7 MHz frequency range using the conventional approach exploiting a sky model of bright point sources as well as the proposed blind calibration method. For the latter, baselines that differed less than 0.2λ at 85.7 MHz were grouped and considered to measure the same spatial frequency, i.e., they were modeled with a single element in the parameter vector $\boldsymbol{\theta}$ in Eq. (2). The gain phase solutions for one polarization of all 46 antennas obtained by the two methods are shown in Fig. 1.

The results clearly show the better robustness of the proposed method to the content of the scene. Figure 2 shows a small sample of scenes at different frequencies. At 22.1 MHz, the image is completely dominated by diffuse emission from the Galactic plane; the point sources are completely invisible, even on the baselines to the calibration dipoles, so conventional calibration does not manage to find meaningful solutions below 30 MHz. At 41.0 MHz there is an interfering source (upper left corner), which conventional calibration confuses with one of the calibration sources. At 41.0 and 50.0 MHz, the two point sources used for calibration (more clearly visible at 65.0 MHz as the blobs in the lower right quarter) are just visible with the 46-antenna array and are clearly discernible using the calibration antennas. A salient feature of the gain phase solutions obtained by conventional calibration (shown in the top panel of Fig. 1) is the wiggle as function of frequency. This is caused by the commonly made but erroneous assumption that all element radiation patterns are identical. This assumption is also made by near redundancy calibration but the latter also exploits the diffuse emission in the scene thus sampling the radiation patterns at far more points. This averages out the variations between the individual radiations patterns thereby reducing their impact.

IV. Conclusions

In this paper, I presented a novel blind calibration technique for tightly-packed irregular arrays. The method exploits the fact that specific nearly-identical frequencies may be sampled multiple times. The method was successfully demonstrated on data from a LOFAR station showing that the proposed method provides significant robustness to variations in the proposed scene. The results also indicate that the errors introduced by modeling nearly-identical spatial frequencies as being exactly identical may even be less detrimental to the solutions than the invalid but commonly made assumption that all element radiation patterns are equal. The proposed method thus seems to provide an attractive approach for calibration of the very compact LFAA stations of the SKA.

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References

[1] P. E. Dewdney, P. J. Hall, R. T. Schilizzi, and T. J. L. W. Lazio, "The Square Kilometre Array," *Proceedings of the IEEE*, vol. 97, no. 8, pp. 1482–1496, Aug. 2009.

[2] M. P. Wylie, S. Roy, and H. Messer, "Joint DOA Estimation and Phase Calibration of Linear Equispaced (LES) Arrays," *IEEE Transactions on Signal Processing*, vol. 42, no. 12, pp. 3449–3459, Dec. 2012.

[3] P. Heidenreich, A. M. Zoubir, and M. Rubsamen, "Joint 2-D DOA Estimation and Phase Calibration for Uniform Rectangular Arrays," *IEEE Transactions on Signal Processing*, vol. 60, no. 9, pp. 4683–4693, Sep. 2012.

[4] S. J. Wijnholds and P. Noorishad, "Statistically Optimal Self-Calibration of Regular Imaging Arrays," in *European Signal Processing Conference (EUSIPCO)*, Bucharest (Romania), 27–31 Aug. 2012, pp. 1304–1308.

[5] J. E. Noordam and A. G. de Bruyn, "High dynamic range mapping of strong sources, with application to 3C84," *Nature*, vol. 299, pp. 597–600, Oct. 1982.

[6] M. P. van Haarlem *et al.*, "LOFAR: The LOw-Frequency ARray," *Astronomy & Astrophysics*, vol. 556, no. A2, pp. 1–53, Aug. 2013.