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Mitigation of Non-Narrowband Radio Frequency Interference

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Abstract

The rapid development and implementation of wireless communication standards put increasing pressure on spectrum allocation and therefore threatens the efficacy of radio astronomy. For example, digital audio broadcasting (DAB) is a wide-bandwidth broadcast technology that is now being implemented and has spectrum allocated in the L band. A DAB signal using a 192 kbit/s codec requires 0.260 MHz per audio programme, which is slightly larger than the standard sub-band width of the Low Frequency Array LOFAR (0.195 MHz). We demonstrate that standard narrowband subspace subtraction methods may provide insufficient suppression of such signals. We therefore propose two algorithms that take into account the non-narrowband nature of these signals. An experimental demonstration of the proposed algorithms both yielded an increase of approximately 2 times in the amount of bandwidth per channel that can be processed when compared to conventional narrowband techniques (for the same attenuation of the RFI signal). The performance of the two methods are identical for LOFAR station configurations with bandwidths between 763 Hz to 195 kHz.

1 Introduction

In the development of radio frequency interference (RFI) mitigation methods, the assumption that the RFI is narrowband, is usually made. If this is the case, spatial RFI mitigation methods such as orthogonal projection, orthogonal projection with subspace bias correction, oblique projection and subspace subtraction [1, 2] can be applied. When the signal is not narrowband, the model for the array response vector becomes a function of bandwidth. The result is that the RFI will appear as an extended source that consists of multiple sources, albeit with rapidly decreasing power. Using a flat frequency response model and Zatman's approximation [4] of that model, we present new subspace subtraction algorithms. To evaluate the proposed RFI mitigation methods the layout of High Band Antenna (HBA) station RS407 in the Low Frequency Array (LOFAR) [3] is used. The HBA stations in LOFAR have an operating band from 110-250 MHz which contains many digital audio broadcasts (DAB).

2 Notation

$\mathbf{A}$	Bold upper-case letters are matrices. The jk^{th} element is indicated by $\mathbf{A}_{jk}$.
$\mathbf{a}$	Bold lower-case letters are column vectors. The j^{th} element is indicated by $\mathbf{a}_j$.
$\mathbf{I}$	Identity matrix.
$\lvert\cdot\rvert$	Absolute value of a scalar.
$\text{Tr}(\cdot)$	Trace of a matrix.
$\text{diag}(\cdot)$	Converts a vector into a diagonal matrix.
i	Square root of -1.
c	Speed of light.
$\{\cdot\}^H$	Hermitian transpose of a matrix.
$\{\cdot\}^T$	Transpose of a matrix.
$\{\cdot\}^*$	Complex conjugate of a scalar.
$\text{sinc}(x)$	$= \sin(\pi x)/(\pi x)$, normalised sinc function.

3 Narrowband Signal Model

If omnidirectional antennas are used, then the normalised array response vector for an array with N_e elements and a continuous wave source with frequency ν is given by $\mathbf{a} = \left[g_1 e^{-\mathrm{i}2\pi\nu\tau_1} \cdots g_{N_\text{e}} e^{-\mathrm{i}2\pi\nu\tau_{N_\text{e}}}\right]^T$. If the source lies in the far-field, then $\tau_j = -(l_\text{s}x_j + m_\text{s}y_j + n_\text{s}z_j)/c$, $g_j = 1/\sqrt{N_\text{e}}$ and x_j, y_j, z_j are the Cartesian coordinates of the j^{th} antenna and l_s, m_s, n_s are directional cosines. The array covariance matrix for a single source without noise is given by $\mathbf{R} = \sigma_\text{s}^2 \mathbf{a}\mathbf{a}^H$ where

$$\mathbf{R}_{jk} = g_j g_k \sigma_\text{s}^2 e^{-\mathrm{i}2\pi\tau_{jk}\nu}, \tag{1}$$

where σ_s^2 is the signal power, $\mathbf{R}_{jk}$ is the jk^{th} element in the covariance matrix and $\tau_{jk} = \tau_j - \tau_k$.

4 Non-Narrowband Signal Model

If the channel bandwidth is not sufficiently narrow, the dependence of the array response vector on frequency becomes significant. The frequency dependent covariance matrix $\mathbf{R}(\nu)$ with only a single interferer (no noise or cosmic sources), that is modelled as a point source, can be written as $\mathbf{R}(\nu) = \sigma_\text{s}^2(\nu)\mathbf{a}(\nu)\mathbf{a}^H(\nu)$, where $\mathbf{a}(\nu)$ is the normalised frequency dependant array response vector. When the different frequency components are uncorrelated, the total covariance matrix is found by integrating over the entire bandwidth $\mathbf{R} = \frac{1}{\Delta\nu}\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \sigma_\text{s}^2(\nu)\mathbf{a}(\nu)\mathbf{a}^H(\nu)d\nu$, where $\Delta\nu$ is the bandwidth and ν_0 is the center frequency. For a flat frequency response, the above integral can be calculated and the jk^{th} element is given by

$$\mathbf{R}_{jk} = \sigma_s^2 g_j g_k \operatorname{sinc}(-\tau_{jk}\Delta\nu) e^{-i2\pi\tau_{jk}\nu_0}. \quad (2)$$

By taking the bandwidth into consideration (and assuming a flat frequency response) this covariance matrix model differs from the narrowband model (see equation (1)) with a sinc function that is dependent on the delay τ_{jk} and the bandwidth $\Delta\nu$. As the bandwidth increases from a single frequency, the sinc function starts to decrease from unity and the effect is that the covariances start to decorrelate. This causes the eigenvalue structure of the array covariance matrix to change. For a single frequency signal there will only be one non-zero eigenvalue. For a non-zero bandwidth signal the covariance matrix will be of full rank, since it is an infinite sum of frequencies. As the bandwidth increases, the largest eigenvalue will decrease and the other eigenvalues will increase. However, most of the eigenvalues will be so small relative to the cosmic sources and the noise in the system, that they can be approximated by zero. The effective rank of the RFI covariance matrix (no noise or cosmic sources) is then defined to be equal to the number of eigenvalues that are significant when compared to the eigenvalues of the covariance matrix that contains only the cosmic sources and noise.

5 Approximation of RFI Eigenvalues

If a covariance matrix has an effective rank of two, it can be approximated by the sum of two discrete uncorrelated signals

$$\mathbf{R} \approx \sigma_1^2 \mathbf{a}_1 \mathbf{a}_1^H + \sigma_2^2 \mathbf{a}_2 \mathbf{a}_2^H. \quad (3)$$

The closed form solution for the eigenvalues of the system given in equation (3) are [2, p. 65]

$$\lambda_{1,2} = \frac{1}{2}(\sigma_1^2 + \sigma_2^2)\left(1 \pm \sqrt{1 - 4\frac{\sigma_1^2\sigma_2^2(1 - |\mathbf{a}_1^H\mathbf{a}_2|^2)}{(\sigma_1^2+\sigma_2^2)^2}}\right). \quad (4)$$

In the model proposed by Zatman the signals are required to have equal power ($\sigma_1^2 = \sigma_2^2 = \sigma^2$) [4]. The equal power criterion is achieved when the discrete sources are arranged in such a way that the instantaneous frequency spectrum mean and variance correspond to the mean and variance of the non-zero bandwidth signal, respectively. Consequently, the distance from the centre frequency ν_0 is given by $\kappa = \frac{\Delta\nu}{2\sqrt{3}}$. Thus, the model in equation (3) becomes

$$\begin{aligned}\mathbf{R} &\approx \sigma^2 \mathbf{a}(\nu_0+\kappa)\mathbf{a}^H(\nu_0+\kappa) + \sigma^2\mathbf{a}(\nu_0-\kappa)\mathbf{a}^H(\nu_0-\kappa)\\ &= \sigma^2(\mathbf{a}_1\mathbf{a}_1^H + \mathbf{a}_2\mathbf{a}_2^H). \quad (5)\end{aligned}$$

Zatman's approach is now generalised from a uniform linear array to an array of any shape using the normalised response vector in equation (1) and the assumption that $\sigma_1^2 = \sigma_2^2 = \sigma^2$. Consequently, equation (4) simplifies to

$$\lambda_{1,2} = \sigma^2[1 \pm |\psi|], \quad (6)$$

where $\psi = \mathbf{a}_1^H \mathbf{a}_2$.

6 Appropriate Bandwidth for Spatial Filtering

Spatial nulling techniques work by modifying eigenvalues in the measured covariance matrix that contain the RFI source. For example, orthogonal projection makes those eigenvalues zero. As the number of eigenvalues that are modified increases so does the loss in information [1]. Therefore, the lowest order filter that sufficiently suppresses the RFI is desired. This criterion can be met by setting the channel bandwidth so that the second eigenvalue lies sufficiently below the noise floor $\lambda_2 << \sigma_n^2\left(1+\sqrt{N_e/N_t}\right)^2$, where N_t is the number of samples used to estimate the array covariance matrix [1]. Increasing the bandwidth so that the second eigenvalue is above the noise and then using second order spatial filtering will not sufficiently remove the RFI. The reason is that the third eigenvalue will then have a significant impact, because no array is perfectly calibrated and the frequency response is not completely flat and this causes the third eigenvalue to be substantially higher than predicted by the model. To increase the bandwidth that can be processed or the RFI suppression, the algorithms that are proposed in section 8 construct a second order filter that does not require the second eigenvalue to be above the noise.

7 Approximating RFI Vector Space

The RFI covariance matrix, in equation (5), can be rewritten in terms of its eigenvalue decomposition $\mathbf{R} \approx \sigma^2(\mathbf{a}_1\mathbf{a}_1^H + \mathbf{a}_2\mathbf{a}_2^H) = \lambda_1\mathbf{v}_1\mathbf{v}_1^H + \lambda_2\mathbf{v}_2\mathbf{v}_2^H$, which maximises the power in the direction of $\mathbf{v}_1$, with the remaining power contained in the direction of $\mathbf{v}_2$. Both vectors are linear combinations of $\mathbf{a}_1$ and $\mathbf{a}_2$,

$$\mathbf{v}_{1,2} = \beta_{1,2}\mathbf{a}_1 + \gamma_{1,2}\mathbf{a}_2, \quad (7)$$

with the following properties: $\mathbf{v}_1^H\mathbf{v}_1 = \mathbf{v}_2^H\mathbf{v}_2 = 1$ (that is unit vectors) and $\mathbf{v}_1^H\mathbf{v}_2 = \mathbf{v}_2^H\mathbf{v}_1 = 0$ (that is $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal). Using the definition of an eigenvector and eigenvalue

$$\begin{aligned}\mathbf{R}\mathbf{v} &\approx \sigma^2(\mathbf{a}_1\mathbf{a}_1^H + \mathbf{a}_2\mathbf{a}_2^H)(\beta\mathbf{a}_1 + \gamma\mathbf{a}_2)\\ &= \sigma^2(\beta\mathbf{a}_1 + \gamma\psi\mathbf{a}_1 + \beta\psi^*\mathbf{a}_2 + \gamma\mathbf{a}_2)\\ &= \lambda\mathbf{v} = \lambda\beta\mathbf{a}_1 + \lambda\gamma\mathbf{a}_2.\end{aligned}$$

Comparing the components of $\mathbf{a}_1$ and $\mathbf{a}_2$, respectively, yields

$$\lambda\beta = \sigma^2(\beta + \gamma\psi), \quad (8)$$

$$\lambda\gamma = \sigma^2(\beta\psi^* + \gamma). \quad (9)$$

Substituting $\lambda = \lambda_{1,2} = \sigma^2(1 \pm |\psi|)$, $\beta = \beta_{1,2}$ and $\gamma = \gamma_{1,2}$ into equations (8) and (9) yields

$$\beta_{1,2} = \pm\frac{\gamma_{1,2}\psi}{|\psi|}, \quad (10)$$

$$\gamma_{1,2} = \pm\frac{\beta_{1,2}\psi^*}{|\psi|}, \quad (11)$$

$$|\beta_{1,2}|^2 = |\gamma_{1,2}|^2. \quad (12)$$

Using the property that $\mathbf{v}$ is a unit vector gives

$$\begin{aligned}\mathbf{v}^H\mathbf{v} = 1 &= (\beta^*\mathbf{a}_1^H + \gamma^*\mathbf{a}_2^H)(\beta\mathbf{a}_1 + \gamma\mathbf{a}_2) \\ &= |\beta|^2 + \beta^*\gamma\psi + \gamma^*\beta\psi^* + |\gamma|^2, \end{aligned} \quad (13)$$

and substituting equations (11) and (12) yields

$$|\beta_{1,2}|^2 = \frac{1}{2(1 \pm |\psi|)}. \quad (14)$$

The phase for either β or γ can be arbitrarily chosen (see equations (10) to (13)). There is also no phase relationship between β_1, γ_1 and β_2, γ_2 as can be seen by expanding $\mathbf{v}_1^H\mathbf{v}_2 = 0$. The phase of $\beta_{1,2}$ is fixed to 0 and thus $\beta_{1,2} = 1/\sqrt{2(1 \pm |\psi|)}$. Substituting equation (11) into equation (7) yields

$$\mathbf{v}_1 = \frac{1}{\sqrt{2(1+|\psi|)}}\left[\mathbf{a}_1 + \frac{\psi^*}{|\psi|}\mathbf{a}_2\right], \quad (15)$$

$$\mathbf{v}_2 = \frac{1}{\sqrt{2(1-|\psi|)}}\left[\mathbf{a}_1 - \frac{\psi^*}{|\psi|}\mathbf{a}_2\right]. \quad (16)$$

To construct $\mathbf{v}_2$ from equation (16) the direction of arrival of the RFI (this can be obtained by using algorithms such as MUSIC and ESPRIT) as well as the signal bandwidth are required.

8 Proposed RFI Mitigation Algorithms

Two new spatial RFI mitigation algorithms based on subspace subtraction are presented in this section. These algorithms are designed for wideband RFI that is stationary, such as DAB broadcasts. The channel bandwidth should be selected so that the second eigenvalue is lower or equal to the power of the cosmic sources being observed. The first algorithm is based on the flat frequency response model (see equation (2)) and the other on Zatman's approximation to that model (see equation (16)). The following preprocessing steps are required:

- Use the eigenvalue decomposition eigen$(\widehat{\mathbf{R}}) = \mathbf{U}\mathbf{S}\mathbf{U}^H$ and let $\widehat{\mathbf{v}}_1 = \mathbf{U}(:,1)$ (where the diagonal entries of $\mathbf{S}$ contain the eigenvalues in decreasing value).
- Obtain the direction of arrival of the RFI source $(\widehat{l}_s, \widehat{m}_s, \widehat{n}_s)$. For example, the location of DAB towers can be easily obtained and used as the initial guess for an algorithm such as Minimum Error Convergence [5]. If no initial guess is known, algorithms such as MUSIC or ESPRIT can be used.
- Estimates for the largest two eigenvalues of the RFI only covarancie matrix can be obtained by using the estimated location of the RFI and equation (6)

$$\widehat{\lambda}_{r,1} = \mathbf{S}(1,1) - \mathrm{Tr}(\mathbf{S}(2:N_e, 2:N_e))/(N_e - 1), \quad (17)$$

$$\widehat{\lambda}_{r,2} = \lambda_{r,1}\left(\frac{1-|\widehat{\psi}|}{1+|\widehat{\psi}|}\right). \quad (18)$$

Use these two new eigenvalue estimates to create the matrix $\widehat{\mathbf{S}}_r = \mathrm{diag}([\widehat{\lambda}_{r,1}\widehat{\lambda}_{r,2}]^T)$.

Algorithm 1: Flat frequency response model based algorithm (FF algorithm)

- Calculate the normalised flat frequency covariance matrix model of the RFI source $\widehat{\mathbf{R}}_f$, using equation (2). Note that this model covariance matrix does not include any noise and that $\sigma_s^2 = 1$.
- Apply eigenvalue decomposition eigen$(\widehat{\mathbf{R}}_f) = \mathbf{U}_f\mathbf{S}_f\mathbf{U}_f^H$ and set $\widehat{\mathbf{v}}_{2,f} = \mathbf{U}(:,2)$.
- Apply subspace subtraction to obtain the flat frequency model based RFI mitigated covariance matrix $\widehat{\mathbf{R}}_{m,f} = \widehat{\mathbf{R}} - [\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_{2,f}]\widehat{\mathbf{S}}_r[\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_{2,f}]^H$.

Algorithm 2: Zatman's approximation based algorithm (ZA algorithm)

- Calculate the normalised Zatman's approximation based model eigenvector $\widehat{\mathbf{v}}_{2,z}$ using equation (16).
- Apply subspace subtraction to obtain the Zatman model based RFI mitigated covariance matrix $\widehat{\mathbf{R}}_{m,z} = \widehat{\mathbf{R}} - [\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_{2,z}]\widehat{\mathbf{S}}_r[\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_{2,z}]^H$.

9 Evaluation of RFI Mitigation Algorithms

To evaluate the performance of both algorithms in section 8, an estimated covariance matrix was created by adding an estimated noise and cosmic source covariance matrix $\widehat{\mathbf{R}}_{nc}$ to an estimated RFI covariance matrix $\widehat{\mathbf{R}}_r$. The matrix $\widehat{\mathbf{R}}_{nc}$ was obtained from a real observation done with LOFAR HBA station RS407 where there is no RFI present. A software defined radio was used to record a DAB signal and has a reasonably flat frequency spectrum. Finite impulse response filters were used to produce 70 signals with bandwidths ranging from 763 Hz to 195 kHz. Each filtered signal was upsampled and frequency shifted to 1 MHz. Covariance matrices were created for each of the 70 processed signals by adding an appropriate delay for each antenna and correlated. To measure the performance of the proposed algorithms the Frobenius norm of the difference between the recovered covariance matrix $\widehat{\mathbf{R}}_m$ and matrix $\widehat{\mathbf{R}}_{nc}$ is used

$$\mathrm{FN} = \sqrt{\sum_{j=1}^{N_e}\sum_{k=1}^{N_e}\left|\widehat{\mathbf{R}}_{nc}(j,k) - \widehat{\mathbf{R}}_m(j,k)\right|} \quad (19)$$

In figure 1 a plot is given of the Frobenius norm as a function of fractional bandwidth. The fractional bandwidth is so small that the difference between the performance of the FF and the performance of the ZA algorithms is less than 10^{-13} and are represented by the proposed 2nd order line. The 1st order line is the performance achieved

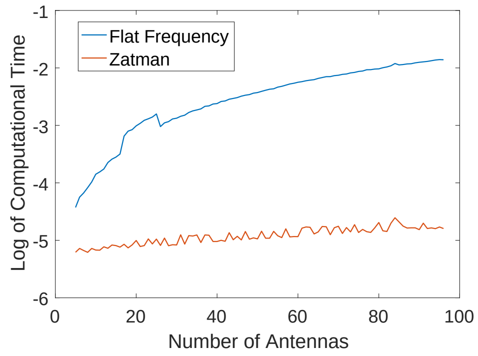

Figure 1. Frobenius norm of the difference between the recovered matrices using RFI mitigation methods and the noise and cosmic source covariance matrix as a function of fractional bandwidth.

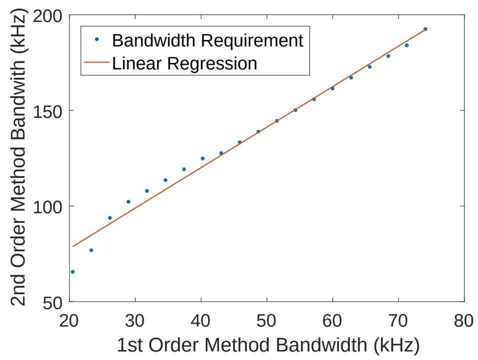

Figure 2. A plot of the bandwidth required by the 2nd order methods as a function of the bandwith of the 1st order method to obtain the same attenuation in bandwidth.

by using single frequency subspace subtraction. Close to zero fractional bandwidth, the performance of the 1st order method and that of the 2nd order methods are the same. As the bandwidth increases so does the Frobenius norm for the 1st order method, since the second eigenvalue becomes significant, however the FN for the 2nd order methods shows very little increase. The minimum achievable FN is just under 0.2 and is due to the estimation errors in $\widehat{\mathbf{v}}_1$ and $\widehat{\lambda}_{r,1}$. Channels with larger bandwidth can be processed using the 2nd order methods, while achieving the same level of mitigation as the 1st order method which requires channels with smaller bandwidth. This is shown in figure 2 where the bandwidth required by the second order methods is given as a function of the bandwidth of the first order method. Using a fitted straight gives an increase of 2 times the amount of bandwidth that can be processed.

The FF algorithm and the ZA algorithm have the same performance for the bandwidths selected. However, the FF algorithm has computational complexity $\mathscr{O}((N_e^2 + N_e)/2)$ while the ZA algorithm only has computational complexity $\mathscr{O}(N_e)$. The FF algorithm requires that an $N_e \times N_e$ matrix be generated of which only $(N_e^2 + N_e)/2$ elements must be calculated because the matrix is Hermitian (see equation (2)). In comparison, the ZA algorithm requires that two N_e-element vectors be calculated and combined using equation (16).

10 Conclusion

Strong wideband RFI cannot be modelled as a single point source, but rather as an infinite sum of sources that rapidly decrease in power. For traditional spatial filtering to work on powerful wideband RFI, it must be filtered into sub-bands which are sufficiently narrow so that for each sub-band the RFI source is a single point source. This greatly increases the computational cost. To reduce this cost, the FF and ZA algorithms are presented that combine a wide-band signal model with a subspace subtraction method and in so doing decreases the number of sub-bands that must be processed. The proposed algorithms are able to process approximately 2 times more bandwidth when compared to conventional spatial filtering methods. The FF algorithm has computational complexity $\mathscr{O}((N_e^2 + N_e)/2)$ while the ZA algorithm only has computational complexity $\mathscr{O}(N_e)$. For the bandwidths between 763 Hz to 195 kHz and using a LOFAR HBA station layout, the performances of the proposed methods are similar.

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References

[1] S. van der Tol and A. van der Veen, “Performance Analysis of Spatial Filtering of RF Interference in Radio Astronomy,” *IEEE Transactions on Signal Processing*, **53**, 3, February 2005, pp. 896–910, doi:10.1109/TSP.2004.842177.

[2] A. J. Boonstra, “Radio Frequency Interference Mitigation in Radio Astronomy,” *PhD Thesis, Delft University of Technology*, June 2005, uuid:caa1942c-4180-4a17-88db-cec359490aad.

[3] M. P. van Haarlem et al., “LOFAR: The Low Frequency Array,” *Astronomy & Astrophysics*, **556**, A2, August 2013, pp. 1–53, doi:10.1051/0004-6361/201220873.

[4] M. Zatman, “How narrow is narrowband?,” *IEE Proceedings-Radar, Sonar and Navigation*, **145**, 2, April 1998, pp. 85–91, doi:10.1049/ip-rsn:19981670.

[5] J. W. Steeb, D. B. Davidson, and S. J. Wijnholds, “Computationally efficient near-field radio frequency source localisation,” *2017 XXXIInd URSI GASS*, August 2017, pp. 1–4, doi: 10.23919/URSI-GASS.2017.8104498.