Performance Improvement of Self-Holography Based Aperture Array Station Calibration

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***Abstract*—The Mid-Frequency Aperture Array (MFAA) of the Square Kilometre Array will consist of around 250 stations (subarrays). Each of these stations will have between 10^3 and 10^4 receive paths. Calibration of the complex valued receive path gains is usually done based on the full array correlation matrix which, at this scale, can be extremely computationally expensive. To solve this problem a self-holography method is suggested in which the complex gains are calculated by correlating the receive paths with a reference signal obtained from the same station. Initial simulation results proved its feasibility, but it was concluded that their is an avenue for improvement in the rate of convergence and the bias level of the estimated gains. This paper explores two methods to remove/ reduce the bias and to improve convergence. Simulations of these two methods are performed and the results are discussed.**

I. Introduction

Parabolic reflector antennas have traditionally been the instrument of choice for radio astronomy since around the 1960's. However, with the ever-increasing demand in survey speed for new science cases, it became apparent that an alternative instrument should be developed that can provide the necessary field of view. Aperture array radio telescopes can provide a large field of view (FoV) thanks to their multi-beam capabilities, where the only limitation is the available processing power and the embedded element pattern of the individual elements in the array. It is therefore a very attractive alternative.

The Mid-Frequency Aperture Array (MFAA) of the Square Kilometre Array (SKA), although still in its concept phase, is envisaged to provide a field of view of between 100 and 200 sq. deg. It is expected that it will consist of around 250 stations each with a diameter of between 40 to 60 m. At station level, the FoV requirement inherently limits the number of antennas that can be combined in the analogue beamforming stage. A smaller number of input signals per analogue beamformer (and therefore more tiles) results in a larger FoV but also increases the required processing power of the station correlator. It is therefore expected that the number of receive paths in an MFAA station will be between 10^3 and 10^4[1][2].

Calibration of the complex valued receive path gains is a key prerequisite to obtain high-quality images. At station level, these are normally calibrated based on the array correlation matrix. However, at this scale, such an approach will likely be infeasible due to the very large number of receive paths. In [3] it is suggested that calibration is done based on a holographic measurement where the receive paths to be calibrated are correlated with a reference signal. This will reduce the required number of correlations from $P(P-1)/2$ to $P+1$ where P are the number of receive paths in the station. The results obtained in [3] prove the feasibility of the concept. However, it concludes that certain improvements can be made on its performance. Two main performance issues are identified as the typical oscillating nature of the estimated gain amplitudes as a function of iteration, and the fact that it converges to a constant offset from the true gains, which indicates a bias. This paper addresses these issues and shows that a significant improvement can be made by carefully modelling the operating environment of the station.

II. Self-Holography Data Model

Since this work is an extension to [3], it was decided to maintain consistency by using the same data model. This section describes the data model.

A station consists of P tiles, each combining M antennas in an analogue beamforming stage. In this way there are P receive paths with unknown complex gains. It is assumed that a calibration source is located in both the tile and reference beam. With the narrow band condition assumed, the geometric delay of the signal at the mth antenna in the pth tile at frequency f is described by the phasor $a_{p,m} = e^{2\pi j \boldsymbol{\xi}_{p,m} \cdot \boldsymbol{l}/\lambda}$, where $\boldsymbol{\xi}_{p,m}$ is the Cartesian coordinate of the mth antenna in the pth tile and l is the direction cosine of the phase reference position. The source is then spatially filtered by applying the weights $w_{p,m} = a_{p,m}$ in the analogue beamforming stage. With the weights stacked in $M \times 1$ vector $\mathbf{w}_p$, the delay phasors in $M \times 1$ vector $\mathbf{a}_p$ and the element noise signals in $M \times 1$ vector $\mathbf{n}_p(t)$, the signal at the pth tile can be expressed as:

$$x_p(t) = g_p \mathbf{w}_p^H (\mathbf{a}_p s(t) + \mathbf{n}_p(t)), \tag{1}$$

where receive path gain for the pth tile is denoted by g_p and the signal from the calibration source denoted by $s(t)$. With $\mathbf{w}_p$ exactly compensating for $\mathbf{a}_p$, Equation 1 can be rewritten as:

$$x_p(t) = g_p M s(t) + \mathbf{w}_p^H \mathbf{n}_p(t). \tag{2}$$

With these signals stacked in $P \times 1$ vector $\mathbf{x}(t)$, the reference beam signal $y(t)$ is formed by applying the weight vector $\mathbf{w}_{\text{ref}}$ as:

$$y(t) = \mathbf{w}_{\text{ref}}^H \mathbf{x}(t). \tag{3}$$

The correlation $\hat{\mathbf{r}}$ of the tile signals with the reference signal is calculated as:

$$\hat{\mathbf{r}} = \mathbf{x}(t)y(t)^H \tag{4}$$

The expected value of $\hat{\mathbf{r}}$ is calculated as:

$$\begin{aligned}\mathbf{r} &= \varepsilon\left\{\mathbf{x}(t)y(t)^H\right\} \\ &= \varepsilon\left\{\mathbf{x}(t)\left(\mathbf{w}_{\text{ref}}^H\mathbf{x}(t)\right)^H\right\} \\ &= \mathbf{g}(\mathbf{g}^H\mathbf{w}_{\text{ref}})M^2\sigma_{\text{cal}} + M\sigma_{\text{n}}\mathbf{w}_{\text{ref}},\end{aligned} \tag{5}$$

where σ_{cal} is the power of the calibration source and σ_{n} is the noise power in an individual antenna. From this it can be seen that the measured correlations are directly proportional to the true gain vector $\mathbf{g}$ if the SNR is high enough. The proportionality constant is related to the power of the reference signal which can be calculated by its autocorrelation as:

$$r_{yy} = \left|\mathbf{w}_{\text{ref}}^H\mathbf{g}\right|^2 M^2\sigma_{\text{cal}} + (\mathbf{w}_{\text{ref}}^H\mathbf{w}_{\text{ref}})M\sigma_{\text{n}}. \tag{6}$$

Dividing Equation 5 with 6 gives:

$$\frac{\mathbf{r}}{r_{yy}} = \frac{\mathbf{g}(\mathbf{g}^H\mathbf{w}_{\text{ref}})M^2\sigma_{\text{cal}} + M\sigma_{\text{n}}\mathbf{w}_{\text{ref}}}{\left|\mathbf{w}_{\text{ref}}^H\mathbf{g}\right|^2 M^2\sigma_{\text{cal}} + (\mathbf{w}_{\text{ref}}^H\mathbf{w}_{\text{ref}})M\sigma_{\text{n}}}. \tag{7}$$

The reference beam weights are chosen as $\mathbf{w}_{\text{ref}} = \mathbf{1} \oslash \mathbf{g}_0/P$, where $\mathbf{g}_0$ is the current best gain estimate. In this way, the true gains can be estimated in an iterative manner.

If $M^2\sigma_{\text{cal}}$ dominates the noise related term in the numerator and the denominator, then $\mathbf{r}/r_{yy}$ will converge towards the true gain value after a number of iterations. Simulation results in [3] show that the gain phase converges toward the true gain phase value, however, the gain amplitude oscillates before it converges to a fixed value, indicating a bias. The next two sections explore methods of addressing these issues.

III. Reducing Oscillation by Averaging

The mean absolute difference in the gain amplitude results obtained in [3] appeared to oscillate while converging to their final value. The oscillation increases as a function of increasing tile size which is due to the decreasing SNR of the signal and noise terms in Equation 7. This was explained to be typical of alternating direction implicit calibration methods when studying the results obtained in [4] and [5].

The Statistically Efficient and Fast Calibration (StEFCal) method described in [4] shows that these oscillations can be reduced by averaging the previous two gain estimates at each iteration. This method was implemented in simulation using Algorithm 1.

A comparison of the original results with that obtained using the averaging method is shown in Figures 1 and 2.

The oscillations have been significantly reduced, especially for lower tile counts. The bias remains, as expected. However,

Algorithm 1 Averaging

> $\mathbf{g}_0 = \mathbf{1}$
> **for** $i = 1$ to $i_{\max}$ **do**
> $\quad \mathbf{w}_{\text{ref}} = 1 \oslash \mathbf{g}_0/P$
> $\quad \mathbf{g} = \mathbf{r}/r_{\text{yy}}$
> $\quad \mathbf{g}_0 = \left(\mathbf{g}^{[i]} - \mathbf{g}^{[i-1]}\right)/2$
> **end for**

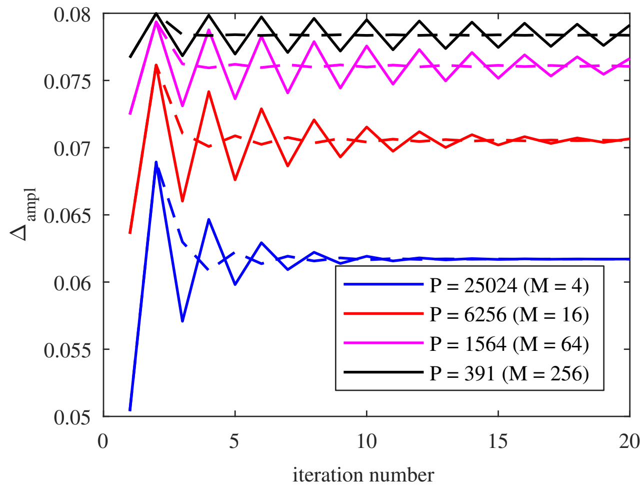

Fig. 1. Comparison of the mean absolute amplitude difference of the true and estimated gains. The previous results obtained in [3] and the corresponding results obtained when using averaging are shown by the solid and dashed lines respectively

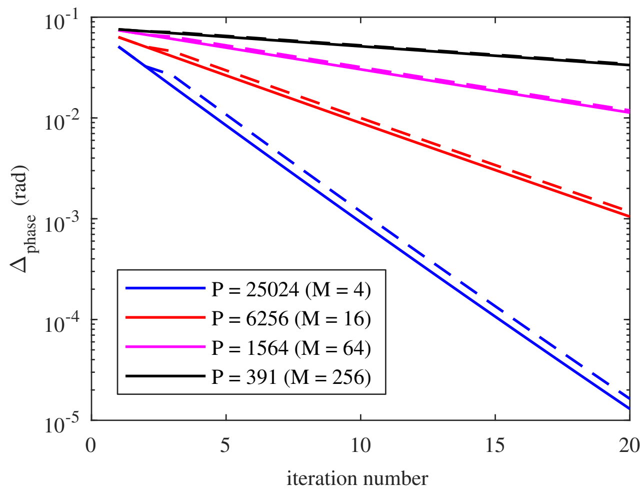

Fig. 2. Comparison of the mean absolute phase difference of the true and estimated gains. The previous results obtained in [3] and the corresponding results obtained when using averaging are shown by the solid and dashed lines respectively

it is seen that the estimated gains converge to a fixed value after at least 10 iterations. The quicker convergence as P decreases can be explained by the relatively constant oscillation amplitude as a function of iteration number for those values of P. The significance of the quicker convergence will be discussed in Section V.

IV. Removing the Noise Term During Measurement

The noise related terms in Equation 7 are a function of only one unknown - the individual element noise power which is defined as $\sigma_{\mathrm{n}} = \sigma_{\mathrm{cal}}/\mathrm{SNR}_{\mathrm{elem}}$. From this it can be seen that the noise related term will increase as a function of a decreasing SNR. Assuming a fixed T_{sys} for all elements, the SNR will solely vary as a function of FoV which is dependent on tile size (and therefore M). The effect of tile size (and FoV) is evident when studying the results in [3].

Phased antenna arrays have the ability to form multiple beams, which, in theory can provide full hemispherical sky coverage. In this case, the noise related terms will be independent of tile size and therefore M. A simulation is performed to illustrate this in more detail. For consistency, the simulation model used is the same as that in [3].

Figures 3 and 4 show the mean absolute difference of the estimated gain amplitude and phase when a hemispherical FoV is considered. As expected, the bias is now almost equal for each value of P. The fact that they are not exactly equal is caused by the fact that, after each iteration, the weights for the reference beam are updated in an attempt to compensate the gain differences between the receive paths while forming the reference beam. As Equation 7 shows, this also affects the noise contribution of the individual receive paths. As a result, the exact relation between the signal and the noise term in both the numerator and the denominator does not only depend on the SNR per element, but also on the level of variation between the receive path gains..

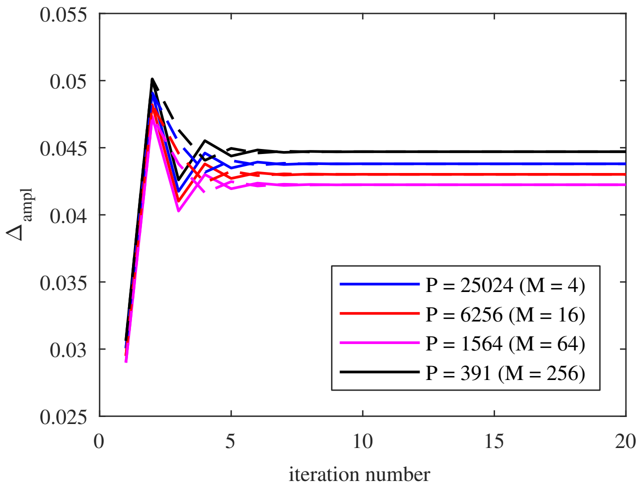

Fig. 3. Mean absolute amplitude difference of the true and estimated gains.

Since the noise power term is dependent on the sky and the operating conditions of the array, it should, in theory, be possible to estimate the noise term given an accurate model of the sky and the array. If the noise power can be estimated accurately, then it can be compensated for by subtracting the noise related term during measurement. This method was tested by simulation using Algorithm 2.

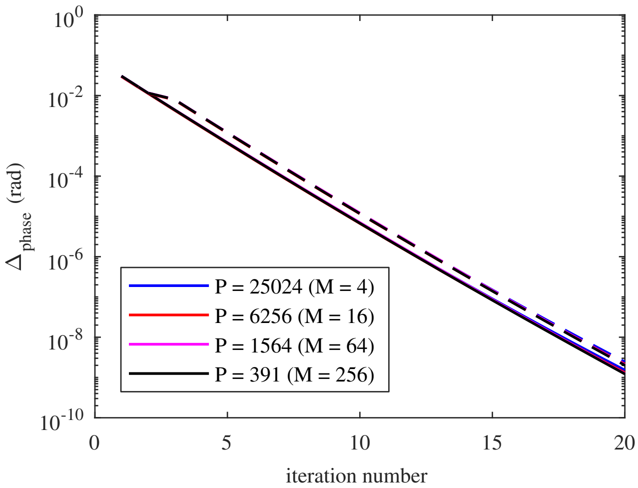

Fig. 4. Mean absolute phase difference of the true and estimated gains.

Algorithm 2 Noise Correcting

$\mathbf{g}_0 = \mathbf{1}$
for $i = 1$ to i_{max} **do**
  $\mathbf{w}_{\mathrm{ref}} = 1 \oslash \mathbf{g}_0 / P$
  $\mathbf{g} = (\mathbf{r} - M\sigma_{\mathrm{n}}\mathbf{w}_{\mathrm{ref}})/r_{\mathrm{yy}}$
  $\mathbf{g}_0 = \mathbf{g}$
end for

The statistical results obtained are shown in Figures 5 and 6.

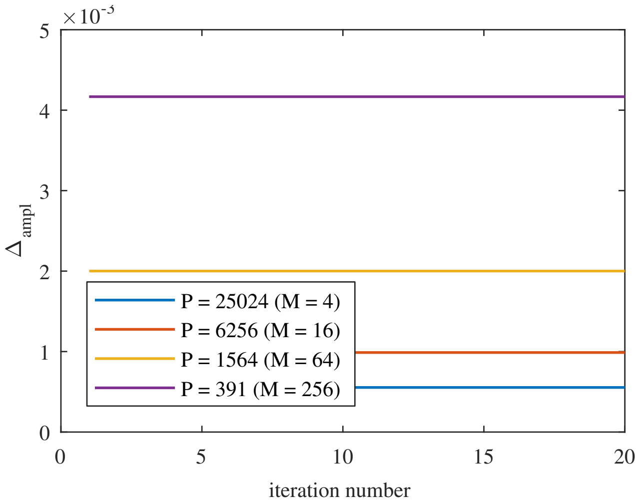

Fig. 5. Mean absolute amplitude difference of the true and estimated gains when compensated for noise during measurement.

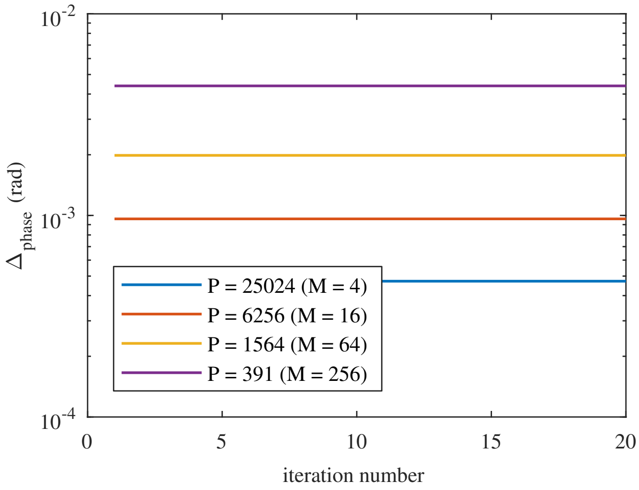

Fig. 6. Mean absolute phase difference of the true and estimated gains when compensated for noise during measurement.

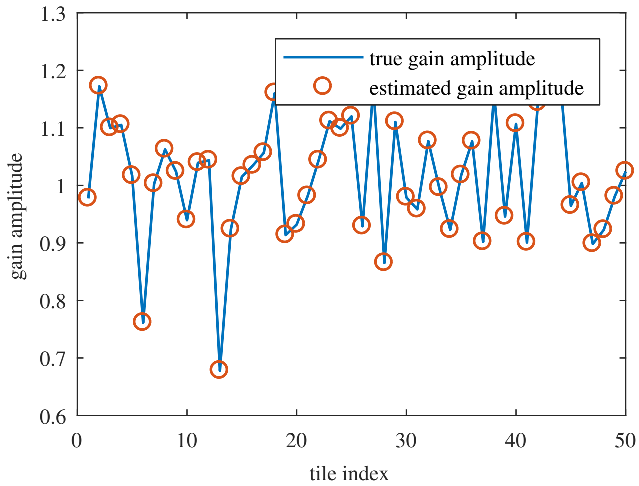

Fig. 7. Comparison of the true and estimated gain amplitudes for the first 100 tiles after the first iteration.

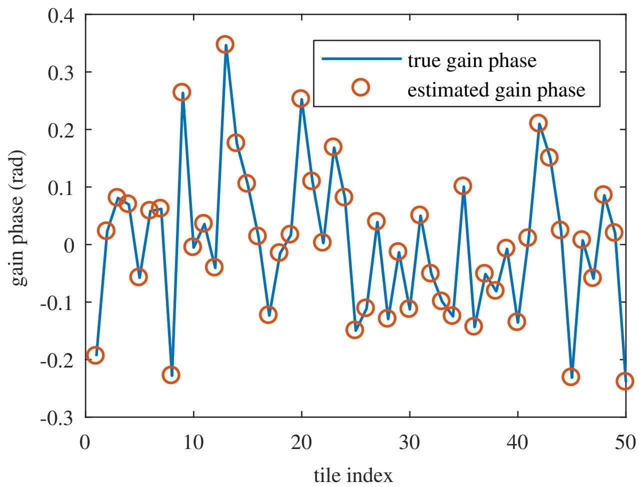

Fig. 8. Comparison of the true and estimated gain phase for the first 100 tiles after the first iteration.

It can be seen that a bias is still present, however, it is significantly lower than in the previous results, indicating that true gains have been more accurately estimated. This is further evident when considering the results for a representative run in Figures 7 and 8. It is also significant to notice that the mean absolute difference between the estimated amplitude and phase does not change by iteration. This can be explained by considering Equation 7 when compensated for noise:

$$\frac{\mathbf{r}}{r_{yy}} = \frac{\mathbf{g}(\mathbf{g}^H \mathbf{w}_{\text{ref}})}{\left|\mathbf{w}_{\text{ref}}^H \mathbf{g}\right|^2}, \tag{8}$$

which can also be expressed as:

$$\frac{\mathbf{r}}{r_{yy}} = \frac{\mathbf{g} \sum_{p=1}^{P} \frac{g_p}{g_{0,p} P}}{\left| \sum_{p=1}^{P} \frac{g_p}{g_{0,p} P} \right|^2} = \mathbf{g} \frac{\alpha}{|\alpha|^2}, \tag{9}$$

where α represents the sum term in the numerator and denominator. If the initial estimated gains are initialised as $\mathbf{g}_0 = \mathbf{1}$, and P is large, then α will approach 1 if the true gain differences are reasonably small, resulting in $\mathbf{g_0} = \mathbf{r}/r_{yy} \approx \mathbf{g}$. Any further iterations will then result in the same value for $\mathbf{g}_0$. The extent of the bias can therefore be directly related to the number of receive paths in the system and their initial gain differences.

V. Discussion of Results

As discussed in [3], standard calibration schemes are based on the full array correlation matrix which requires $4P^2\Delta f$ operations, where Δf is the bandwidth in consideration [6]. The self-holography method reduces this number to $N_{\text{iter}}\Delta f(12P + 4)$ [3] where N_{iter} is the number of iterations required for the estimated gains to adequately converge to the true gains.

By applying the averaging method during iterations as discussed in Section III, it was seen that the number of iterations required to converge to the true gains is significantly reduced, especially for large tile counts. It is therefore a remarkable improvement to the basic self-holography method since the number of required operations and measurements have been reduced even further. A trade-off between the number of receiver paths and the rate of convergence is evident. These results, therefore, additionally provide a guideline from which the calibration performance can be determined given a set of design constraints.

In Section IV it was shown that, theoretically, the noise related term can be compensated for during measurement. A simulation was performed and the results showed that the true gains are estimated fairly accurately after the first iteration and that no further iterations are required. In terms of number operations, this is the largest achievable reduction. It was, however, discussed that a very accurate estimation of the noise power of individual antennas will be required. A quick investigation of this issue showed that even a small compensation error can result in a significant error between the estimated and true gain values. Furthermore, a hemispherical FoV was considered which ignores the effect of tile size. In reality, however, the system is normally constrained in processing power and the embedded element patterns of individual antennas in the array, which significantly affects the FoV. It is concluded that these facts should be taken into account in future studies to determine its impact.

VI. Conclusion

This paper explored two methods with which the performance of station-level calibration using self-holography can be improved. The first method aimed to reduce the oscillation of the estimated gains by averaging the estimation results from the last two iterations. The results showed that the oscillation is significantly lowered, which reduces the number of required iterations to converge to the true gain values. The second method aimed to remove, or at least reduce, the bias observed in the gain estimates during normal operation. This was done by assuming that the noise related terms can be exactly predicted based on knowledge of the operating environment of the system and its capabilities. The results showed that the bias is reduced and that the true gains are almost exactly estimated after the first iteration. It is concluded that this will result in the lowest possible number of required iterations. However, it does require a very precise compensation of the noise related term. This can be quite challenging in practice which suggests that this issue should be explored in future work. Lastly, it should be noted that without noise compensation the phase estimate error reduces to zero if sufficient iterations are made. This shows that only a small, in many cases even negligible, bias in amplitude is unavoidable in the self-holography calibration method.

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